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Reflection of soft x-rays and extreme ultraviolet from a metallic Fibonacci quasi-superlattice

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Abstract. The reflectivity of TE soft x-rays and extreme ultraviolet from a metallic Fibonacci quasi-superlattice is investigated. Based on the hydrodynamic model and by using the transfer matrix method we obtained a formal expression of the reflectivity for the case of a metallic multilayer in the form of a Fibonacci sequence. The numerical results for reflectivities are shown for a set of multilayers with finite generation number. Besides the interesting self-similar pattern of the reflectance peaks, the numbers of which are arranged just corresponding the Fibonacci sequence, we find that the stronger reflectance peak larger height and width moves to a higher-frequency region compared with the usual standard superlattice, which stimulates interest in the study and making of soft x-rays and extreme ultraviolet reflectors.

1. Introduction

A quasi-superlattice is a kind of artificial one-dimensional quasi-crystal which came into being only a few years ago (Merlin *et al* 1985). It possesses quasi-periodicity in one direction, and is homogeneous or periodic in the others. As the solid materials are ordered quasi-periodically, the translational symmetries are then broken and its structural characteristics lie between those of crystals and amorphous solids. Since the quasi-superlattice has some novel properties, it has drawn great attention (Kohmoto *et al* 1987, Peyraud *et al* 1988, Eric Yang *et al* 1988, Ashraff and Stinchcombe 1988, Ma and Tsai 1986). In recent years, a great many publications have been devoted to semiconductor quasi-superlattices (Hawrylak and Quinn 1986, Ma and Tsai 1987, Qin 1988), while for the metallic quasi-superlattice, only a few authors have reported the results of their research work either experimentally or theoretically (Hu *et al* 1986, Karkut *et al* 1986, Xue and Tsai 1988, Yamamoto and Hiragu 1988). Recently, many experimentalists have become very interested in designing and manufacturing mirrors of electromagnetic waves with frequencies ranging from ultraviolets to soft x-rays (for instance, Barbee *et al* 1985, Spiller and Rosenbluth 1985, 1986), because theoretical and experimental researches have revealed that metallic superlattices can be made into reflectors of high-frequency electromagnetic waves. In the hydrodynamic approximation, Xue and Tsai

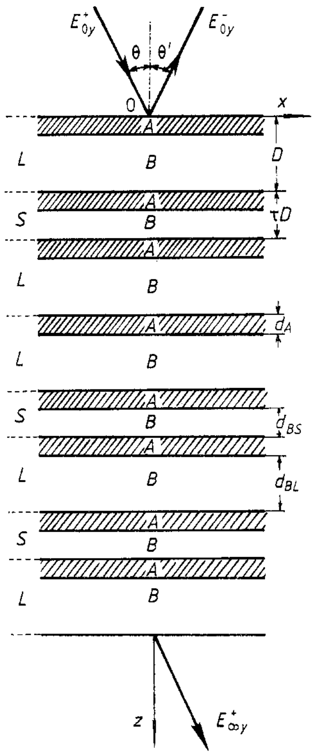

Figure 1. The structure of a metallic Fibonacci quasi-superlattice with 16 layers. E_{0y}^{+}, E_{0y}^{-} and $E_{\infty y}^{+}$ are incident, reflecting and transmission fields, and θ and θ' are incident and reflecting angles respectively.

(1988) presented a new transfer matrix method for the calculation of the reflectivity of TE waves off a metallic superlattice. Feng *et al* (1988) extended the method to the general cases of the disordered metallic multilayers and obtained a formal analytical expression for the reflectivity. In this paper we will apply the previous theory to calculate the reflectivity of the quasi-superlattice. Sections 2 and 3 present the models and methods we utilised and § 4 is devoted to the description and discussion of the numerical results.

2. Equations for the reflectivity of TE waves on a metallic Fibonacci quasi-superlattice

The model of the Fibonacci quasi-superlattice was first presented by Merlin *et al* (1985). Let two types of metallic layers form two elementary seeds as blocks L and S. Block L consists of metal A of thickness d_A and metal B of thickness d_{BL}, while block S is composed of metal A with the same thickness d_A as in block L and the metal B of thickness d_{BS}. The ratio of the thicknesses of the two blocks L and S satisfies $(d_A + d_{BS})/(d_A + d_{BL}) = \tau$, which is the inverse of the golden mean, $\tau = (\sqrt{5} - 1)/2$. The generating rule of blocks L and S is a Fibonacci sequence, i.e.,

$$S_1 = \{L\}, S_2 = \{LS\}, S_3 = \{LSL\}, \ldots, S_n = S_{n-1}S_{n-2}. \qquad (1a)$$

For the nth Fibonacci quasi-superlattice, we have got $N = 2F_n$ metal layers, where F_n is the nth order of the Fibonacci sequence, which is defined as

$$F_0 = 1, F_1 = 1, F_2 = 2, F_3 = 3, F_4 = 5, \ldots, F_n = F_{n-1} + F_{n-2}. \qquad (1b)$$

Let $D = d_A + d_{BL}$, then the total thickness of the multilayer is

$$z_N = (F_{n-1} + \tau F_{n-2})D. \qquad (1c)$$

Figure 1 shows a sample of a fifth-generation metallic Fibonacci quasi-superlattice with 16 layers. For the simple case, we suppose the regions $z < 0$ and $z > z_N$ are vacuum spaces, the incident soft x-rays or the extreme ultraviolet are TE waves with the same frequency and the wavevectors lie on the x–z plane with a common x component q. The z component $\pm k_0$ of a wavevector is determined by

$$k_0^2 = (\omega/c)^2 - q^2 \tag{2a}$$

with

$$q = (\omega/c)\sin\theta \tag{2b}$$

where c is the speed of light in vacuum and θ is the incident angle. For the electric field of TE waves, which has only a y component, we have

$$E_{0y}(q,\omega,z) = E_{0y}^{+}(q,\omega)\exp(\mathrm{i}k_0\cdot z) + E_{0y}^{-}(q,\omega)\exp(-\mathrm{i}k_0\cdot z) \qquad z<0 \tag{3}$$

and for the outgoing waves off the nth Fibonacci metallic multilayers,

$$E_{\infty y}(q,\omega,z) = E_{\infty y}^{+}(q,\omega)\exp(\mathrm{i}k_0\cdot z) \qquad z>z_N. \tag{4}$$

In the case of the metallic Fibonacci quasi-superlattice, we consider the electrons in the multilayers as an ideal liquid, and use the hydrodynamic model to describe the motion of the electrons. By neglecting the damping effects and space dispersions and combining Maxwell's equations with the hydrodynamic equations (Xue and Tsai 1985, 1986, 1988), we obtain an equation for the TE field as

$$\mathrm{d}^2E_y/\mathrm{d}z^2 + [(\omega/c)^2 - (\omega_\mathrm{P}/c)^2 - q^2]E_y = 0. \tag{5}$$

For the two kinds of layered metals within an nth generation Fibonacci quasi-superlattice, we define the plasma frequency ω_P as

$$\omega_{\mathrm{P}A} = [(4\pi n_A e^2/m)]^{1/2} \tag{6a}$$

and

$$\omega_{\mathrm{P}B} = [(4\pi n_B e^2/m)]^{1/2} = \omega_{\mathrm{P}BL} = \omega_{\mathrm{P}BS} \tag{6b}$$

for the metals A and B respectively, where n_A and n_B are just the electron densities in metals A and B.

The general solution for E_y in the multilayers is

$$E_{yj}(q,\omega,z) = E_{yj}^{+}\exp[\mathrm{i}k_j\cdot(z-z_j)] + ET_{yj}^{-}\exp[-\mathrm{i}k_j\cdot(z-z_j)] \tag{7}$$

where E_{yj}^{+} and E_{yj}^{-} are independent of z, and

$$z_j = d_1 + d_2 + \ldots + d_j \tag{8a}$$

$$k_j^2 = k_0^2 - (\omega_{\mathrm{P}j}/c)^2 \tag{8b}$$

here $\omega_{\mathrm{P}j}$ only take the two values as in equations (6*a*) and (6*b*), and

$$j = 1,2,\ldots,N \qquad N = 2F_n. \tag{9}$$

As we are only interested in the region of soft x-rays and ultraviolet, the frequencies of the incident waves in this region could satisfy the following condition:

$$\omega/\omega_{\mathrm{P\,min}} > \sec\theta, \qquad \omega_{\mathrm{P\,min}}\{\omega_{\mathrm{P}A},\omega_{\mathrm{P}B}\}. \tag{10}$$

Let

$$Y = E_{0y}^{-}/E_{0y}^{+} \tag{11}$$

$$U_j = \begin{bmatrix} u_j \\ v_j \end{bmatrix} = \begin{bmatrix} E_{yj}^{+}/E_{0y}^{+} \\ E_{yj}^{-}/E_{0y}^{+} \end{bmatrix} \qquad j = 1,2,\ldots,N \tag{12a}$$

$$\rho_\mu = \exp(\mathrm{i}k_\mu \cdot d_\mu) \qquad \mu = A, BL, BS \qquad k_B = k_{BL} = k_{BS} \tag{12b}$$

and use the continuity condition of E_y and $\mathrm{d}E_y/\mathrm{d}z$ for the two surfaces, we have

$$\begin{bmatrix} \rho_A^{-1} & \rho_A \\ \rho_A^{-1} & -\rho_A \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} 1+Y \\ (1-Y)k_0/k_A \end{bmatrix} \qquad \text{at } z = 0, \tag{13}$$

and

$$(k_0 - k_B)u_N + (k_0 + k_B)v_N = 0 \qquad \text{at } z = z_N \tag{14}$$

while, for all the interfaces, we find a hierarchy for the nth Fibonacci quasi-superlattice:

$$U_1 \begin{cases} = X_A^{-1} T_\mathrm{o} \tilde{X}_{BL} U_N & \text{when } n = \text{odd number} \\ = X_A^{-1} T_\mathrm{e} \tilde{X}_{BS} U_N & \text{when } n = \text{even number} \end{cases} \tag{15}$$

where

$$X_\mu = \begin{bmatrix} 1 & 1 \\ \mathrm{i}k_\mu & -\mathrm{i}k_\mu \end{bmatrix} \qquad \tilde{X}_\mu = \begin{bmatrix} \rho_\mu^{-1} & \rho_\mu \\ \mathrm{i}k_\mu \rho_\mu^{-1} & -\mathrm{i}k_\mu \rho_\mu \end{bmatrix} \qquad \mu = A, BL, BS, \tag{16}$$

and

$$T_\mathrm{o} = C_n C_1^{-1} \qquad \text{when } n = \text{odd number} \tag{17a}$$

$$T_\mathrm{e} = C_n C_0^{-1} \qquad \text{when } n = \text{even number.} \tag{17b}$$

The nth transfer matrix C_n is just defined in the manner of the Fibonacci sequence,

$$C_n = C_{n-1} C_{n-2} \tag{18}$$

with the initial conditions

$$C_0 = M_{BS} M_A \tag{19}$$

$$C_1 = M_{BL} M_A \tag{20}$$

$$M_\mu = \tilde{X}_\mu X_\mu^{-1} \qquad \mu = A, BL, BS. \tag{21}$$

Since equations (13), (14) and (15) form a closed set of equations to determine u_1, v_1, u_N, v_N and Y, thus the reflectivity defined as

$$R = |Y|^2 \tag{22}$$

can be obtained directly by solving the equations.

3. Formal expression for the reflectivity

It is important to note that for any incident wave frequency, the introduced matrix M_μ is always real. For $k_\mu^2 > 0$ in equation (21), we find

$$M_\mu = \begin{bmatrix} \cos k_\mu d_\mu & -\sin k_\mu d_\mu / k_\mu \\ k_\mu \sin k_\mu d_\mu & \cos k_\mu d_\mu \end{bmatrix} \tag{23}$$

while for $k_\mu^2 < 0$,

$$M_\mu = \begin{bmatrix} \cosh \tilde{k}_\mu d_\mu & -\sinh \tilde{k}_\mu d_\mu / \tilde{k}_\mu \\ -\tilde{k}_\mu \sinh \tilde{k}_\mu d_\mu & \cosh \tilde{k}_\mu d_\mu \end{bmatrix} \tag{24}$$

where

$$\mathrm{i}k_\mu = \tilde{k}_\mu \qquad \mu = A, BL, BS. \tag{25}$$

The matrix T defined by equation (17) is, therefore, also real. Let us formally denote it as

$$T = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}. \tag{26}$$

Combining equation (15) with equations (13) and (14), and by making use of the above notation, we finally obtain an analytical expression of the reflectivity for the soft x-rays and extreme ultraviolet on the metallic Fibonacci quasi-superlattice,

$$R = |Y|^2 = (x_1^2 + y_1^2)/(x_2^2 + y_2^2) \qquad (27)$$

where

$$x_1 = -(k_A - k_0)a_1 + (k_A + k_0)a_3 \qquad y_1 = -(k_A - k_0)a_2 + (k_A + k_0)a_4 \qquad (28a)$$

$$x_2 = (k_A + k_0)a_1 - (k_A - k_0)a_3 \qquad y_2 = (k_A + k_0)a_2 - (k_A - k_0)a_4. \qquad (28b)$$

For the case of $\omega > \max\{\omega_{PA}, \omega_{PB}\}$, that is $k_\mu^2 > 0$, we have

$$a_1 = (k_B + k_0)W_1 + (k_B - k_0)W_3 \qquad (29a)$$

$$a_2 = (k_B + k_0)W_2 + (k_B - k_0)W_4 \qquad (29b)$$

$$a_3 = b_1 \cos 2k_A d_A + b_2 \sin 2k_A d_A \qquad (29c)$$

$$a_4 = -b_2 \cos 2k_A d_A + b_1 \sin 2k_A d_A \qquad (29d)$$

and

$$b_1 = (k_B - k_0)W_1 + (k_B + k_0)W_3 \qquad (30a)$$

$$b_2 = (k_B - k_0)W_2 + (k_B + k_0)W_4 \qquad (30b)$$

where

$$W_1 = (\alpha + \beta)\cos k_B d_N + (\gamma - \delta)\sin k_B d_N \qquad (31a)$$

$$W_2 = (\gamma - \delta)\cos k_B d_N - (\alpha + \beta)\sin k_B d_N \qquad (31b)$$

$$W_3 = (\alpha - \beta)\cos k_B d_N + (\gamma + \delta)\sin k_B d_N \qquad (31c)$$

$$W_4 = -(\gamma + \delta)\cos k_B d_N + (\alpha - \beta)\sin k_B d_N \qquad (31d)$$

with

$$\alpha = t_{11}/2 \qquad \beta = t_{22}k_B/2k_A \qquad (32a)$$

$$\gamma = t_{12}k_B/2 \qquad \delta = t_{21}/2k_A \qquad (32b)$$

and

$$d_N \begin{cases} = d_{BL} & \text{when } n = \text{odd number} \\ = d_{BS} & \text{when } n = \text{even number.} \end{cases} \qquad (33)$$

For the case of $\omega_{PA} < \omega < \omega_{PB}$, that is $k_A^2 > 0$, $k_B^2 < 0$, equations (29*a*), (29*b*), (30*a*) and (30*b*) are changed into the following equations,

$$a_1 = (\tilde{W}_1 - \tilde{W}_3)k_0 - (\tilde{W}_2 + \tilde{W}_4)\tilde{k}_B \qquad (34a)$$

$$a_2 = (-\tilde{W}_2 + \tilde{W}_4)k_0 - (\tilde{W}_1 + \tilde{W}_3)\tilde{k}_B \qquad (34b)$$

$$b_1 = (\tilde{W}_1 - \tilde{W}_3)k_0 + (\tilde{W}_2 + \tilde{W}_4)\tilde{k}_B \qquad (35a)$$

$$b_2 = (-\tilde{W}_2 + \tilde{W}_4)k_0 + (\tilde{W}_1 + \tilde{W}_3)\tilde{k}_B \qquad (35b)$$

where

$$\tilde{W}_1 = (\alpha + \gamma)\exp(-\tilde{k}_B d_N) \qquad (36a)$$

$$\tilde{W}_2 = (\delta + \beta)\exp(-\tilde{k}_B d_N) \qquad (36b)$$

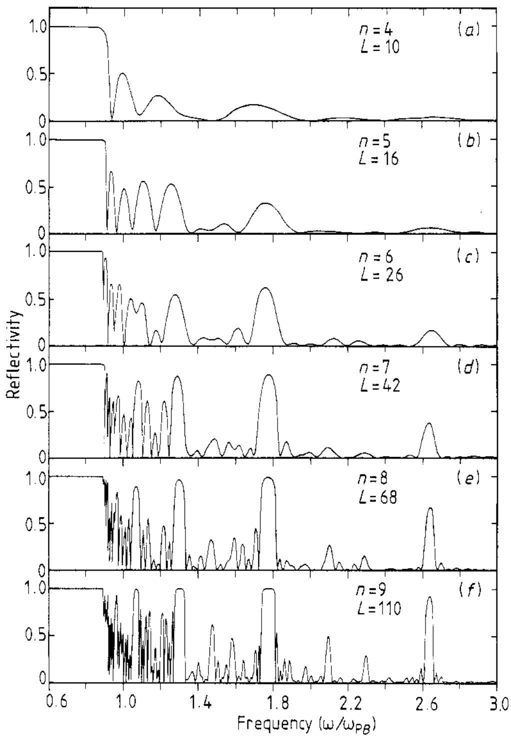

Figure 2. Reflectivities for normal incidence ($q = 0$) on metallic Fibonacci quasi-superlattices for six generations from 4 to 9. L is the metal film layer number. ω_{PB} is the plasma frequency for metal B corresponding to the electron density $n_B = 18.1 \times 10^{23}\ \mathrm{cm}^{-3}$ and the thicknesses for elementary metal films are $d_A = 100$ Å, $d_{BL} = 200$ Å.

$$\tilde{W}_3 = (\alpha - \gamma)\exp(\tilde{k}_B d_N) \tag{36c}$$

$$\tilde{W}_4 = (\delta - \beta)\exp(\tilde{k}_B d_N) \tag{36d}$$

with

$$\tilde{k}_B = \mathrm{i}k_B. \tag{37}$$

From the above equations, one can easily find that at the limiting condition of very high frequency, the reflectance in equation (31) will approach zero in an oscillating manner. On the other hand, when $\omega < \omega_{PA}$, the reflectivity will approach one.

4. Numerical results and discussions

With the formulae presented in §§ 2 and 3, and by using a microcomputer, one can obtain the reflectivity numerically. In our calculation, we take the relevant parameters as $d_A = 100$ Å, $d_{BL} = 200$ Å, $n_A = 8.63 \times 10^{23}\ \mathrm{cm}^{-3}$ and $n_B = 18.1 \times 10^{23}\ \mathrm{cm}^{-3}$. In the frequency range from $0.6\ \omega_{PB}$ to $3\omega_{PB}$ (the corresponding wavelength for free EM waves ranged from 1308 to 262 Å), we plotted the calculated reflectance curves of normal incidence ($q = 0$) for six generations ($n = 4, 5, \ldots, 9$) as shown in figure 2.

From figure 2, we find that the reflectance values stagger down with increasing frequency for the nth metallic quasi-superlattice. There is an overall tendency of decline

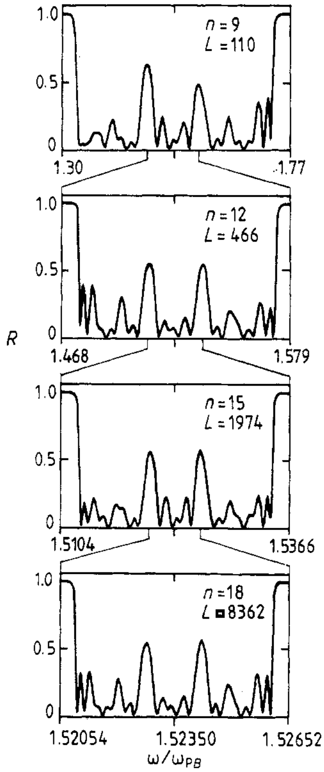

Figure 3. Reflectivity for normal incident waves of the metallic Fibonacci quasi-superlattices of S_n ($n = 9$, 12, 15, 18) around $\omega = 1.5235\omega_{PB}$. The relevant parameters are the same as in figure 2. Note the difference of the scale of ω among the four curves.

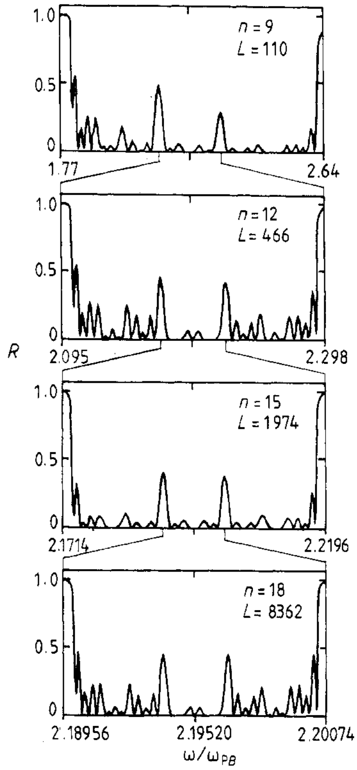

Figure 4. The same as figure 3 around $\omega = 2.1952\ \omega_{PB}$.

of R with ω but the reflectivities fluctuate wildly around the average tendency in a typical quasi-periodic manner compared with the reflection results of the usual periodic AB structure superlattice (Xue and Tsai 1988, Feng *et al* 1988). In the case of a periodic superlattice, we know that the reflection curves also have the tendency to decline, but they are modulated by the periodic principal maxima. In every two neighbouring main peaks, there are $L - 2$ (L is the modulation period number) submaxima. In the case of a quasi-superlattice, however, the pattern of the reflectivity is rearranged with a self-similar structure as n increases. When we pick up any two nearby first-order maxima, for example, we can distinguish the second-order maxima between them, but in turn several third-order maxima again appear between every neighbouring second-order maxima, . . . , and so on.

Recently, Ma and Tsai (1988) investigated interface polariton modes in a semiconductor quasi-superlattice. The polariton spectrum has forbidden bands and allowed bands, the structure of which is very similar to our reflection results if we consider our reflectance nth order maxima as the nth order forbidden frequency bands and the nth order minima as the nth order allowed bands. So our results are easily understood by Ma and Tsai's theory.

For the frequency region $\omega > \omega_{PA}$, one can find infinitely many special points, around which the reflectivity has scaling properties, which is very similar to the situation in optical quasi-periodic media (Kohmoto *et al* 1987). In figures 3 and 4, we have plotted the reflectivity against the frequency around two prominent points $1.5235\omega_{PB}$ and $2.1952\omega_{PB}$ with different generation numbers, and we find that the rescaled reflectance figure of S_{n+6} is similar to that of S_n for any generation number n, which gives us the hint that the introduced transfer matrix C_n has the property of a six-cycle around the fixed points (Kohmoto and Oono 1984, Kohmoto and Banavar 1986, Kohmoto *et al* 1987), that is

$$C_{n+6} = C_n. \qquad (38)$$

In our case, the scale change of the frequency between the plots (see figures 3 and 4) for S_n and S_{n+6} is τ^6 about any scaling points. This peculiar phenomenon resulted from the model we used as we know from the one-dimensional quasi-periodic Schrödinger equation (Kohmoto *et al* 1983, Ostlund *et al* 1983).

The very interesting consequent result is that compared with the periodic superlattice several new peaks appear in the region of high frequency $\omega > 2\omega_{PB}$, which suggests a new way to make and study the mirror for soft x-rays and ultraviolet.

In this paper, we have presented a direct analytical approach to calculate the reflectivity of the TE soft x-rays and ultraviolet on a metallic Fibonacci quasi-superlattice in the hydrodynamic approximation. By using a microcomputer and the scaling properties, one can easily obtain the reflectance results for any designed Fibonacci multilayer. We expect the corresponding experiment to test the prediction of this paper.

In passing we point out that the present theory is based on the hydrodynamic model. We have neglected the possible polarisation effects of the lattice ions in the background. This is only reliable when the energies of the incident waves are not enough to excite the bound electrons on the ions. We have also neglected the damping effects and spatial dispersion. Despite all these disadvantages, the present theory has provided a simple new method to illustrate many new properties of metallic quasi-superlattice and a new way to design metallic multilayers as soft x-ray and ultraviolet reflectors.

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